

Surface Water Flow Simulation using Cellular Automata Based Flow Direction D-Infinity Algorithm

Ishak Muhajir

Department of Mathematics
Fac. Mathematics and Natural Sciences
Hasanuddin University,
Makassar, Indonesia 90245
E-mail: im991@icloud.com

Armin Lawi

Department of Computer Science
Fac. Mathematics and Natural Sciences
Hasanuddin University,
Makassar, Indonesia 90245
E-mail: armin@unhas.ac.id

Agustinus Ribal

Department of Mathematics,
Fac. Mathematics and Natural Sciences
Hasanuddin University,
Makassar, Indonesia, 90245
E-mail: agus.ribal@gmail.com

Abstract—Simulation of surface water flow or runoff is one of the most important thing in order to understand a natural phenomenon when a public policy is under consideration. Generally, runoff simulation is only limited to one flow result thus it is difficult to make a decision. This limitation can be resolved by doing some iterations to get more accurate results. Repeated iterations can be conducted using cellular automata method combined with the flow direction D-infinity algorithm as the transitions. The simulation results in this paper gives a continuous simulation appropriately according to the number of desired iterations. Based on the simulation results, the surface water flow can be determined and therefore the utilization of the quantity of rainwater for various purposes can be used in any areas.

Keywords—cellular automata; flow direction D-infinity algorithm; surface water flow; digital elevation model

I. INTRODUCTION

Surface water flow or runoff is familiar to be observed and there are many scientists have donated their time in reviewing various theories to describe this phenomenon in digital form. Many methods have advantages and disadvantages. The method for calculating runoff for one iteration is not enough and it causes dilemma fairly large. This due to the difficulty of reading a water flow with an irregular surface so that the water flow only move on neighboring pixels of reviews and cannot grow.

Processing of digital elevation models becomes the purpose of many methods to be applied such as in the simple D8 method (eight flow directions). One-way flow method is formed from the deepest of eight neighboring pixels with each neighboring pixel angular width is 45 degrees [1]. The weakness of this method is that it does not give an opportunity to other neighboring pixel to get water flow. Disadvantage of D8 method is resolved by the method of Multiple Flow Direction [2,3]. This flow direction method leads to every lower pixel of the review pixels by dividing each balanced pixel according to the depth of neighboring pixels. Dispersion to more than one pixel does not only apply on this methods alone. In fact, almost every method gives also the same thing such as in Lea's Method and DEMON (Digital Elevation

Model Networks). However, excessive dispersion causes inconsistency toward physical definition of upslope and give the idea to Tarboton to introduce a new method Flow Direction D-Infinity (D_{∞}) [4,5,1].

In this paper a surface water flow simulation is developed using cellular automata method in which the iteration functions (or transitions) use the flow direction D_{∞} . The flow direction in this method leads to an angle between 0 to 2π without a spreading flow [1]. Direction angle is determined by selecting a facet steepest calculated from eight facet neighbors started from facet east the counter clockwise [1]. However, this method as well as some other methods has also several limitations. One of those is that the flow just moves in one iteration and thus it results that the method is not applied when rain falls continuously for long time. Some studies examine this phenomenon of rain water as the most essential phenomenon with large amounts of water. Therefore, we present also the surface water flow simulation when large amount of water fill the cells continuously using cellular automata based flow direction D-infinity algorithm.

II. FUNDAMENTAL CONCEPTS

A. Surface Water Flow

Surface Water Flow is a part of the rainfall that runs off the land surface and take the soil particles [6]. Surface flow direction prediction can be used by institutions to develop various benefits for people in the region. Lack of water for agricultural, plantation, water source and others can be resolved by prediction of the flow direction.

B. Digital Elevation Models

Digital Elevation Models (DEMs) is a digital model or three-dimensional representation of the land surface (generally for a planet like earth, moon, or asteroid). It is made of land elevation data. It is presented with square grid where the value of land elevation is every pixel on the grid.

DEM raster is grid structure which each cell grid have a unique value and the DEM elevation vector is a Triangular Irregular Network (TIN), which consists a set of point

coordinates x, y, z interconnected. DEM raster and TIN are different DEM kind [7].

C. Flow Direction D-Infinity

Tarboton [1] proposed a new method of flow direction called Flow Direction D-Infinity. Term of D-Infinity is used because the direction of flow is determined on a pixel by taking a value between 0 to 2π . This method examines eight triangles terms (eight triangular Facets) for each pixel. Each Facets have a point at the center pixel and two vertex at the center of its neighboring pixels.

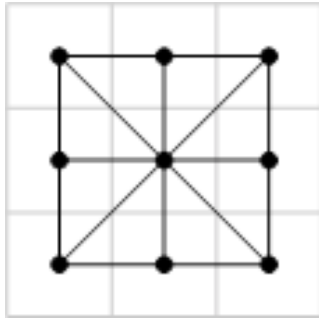


Fig. 1. Eight Triangular Facets

Figure 2 shows a facet east-northeast, where e_0 is the elevation at the center pixel value, e_1 is a value elevation of the eastern part of neighboring pixels, and e_2 is the value of neighboring pixels elevation northeastern, as for d_1 and d_2 is the distance between the centers of the pixels. While flow direction D-infinity focus on the value of the deepest facet. From that facet, it will be examined slope angle from the east clockwise.

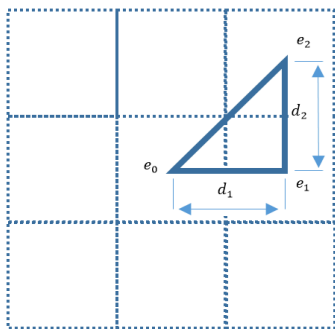


Fig. 2. Facet East-Northeast

Figure 3 shows the eight triangular facets constructed with a midpoint (P_1 and P_2) of two adjacent pixels. The direction of flow pointing between P_1 and P_2 is distributed into two cells based on the direction the slope is steep (i.e. part of the flow to two cells with P_1 and P_2 are respectively $\alpha_1/45^\circ$ and $\alpha_2/45^\circ$. If the angle slope greater than or equal to 45° then the flow direction is on a diagonal line P_2 and if the slope angle is smaller than or equal to 0 then the flow direction is on the line of cardinals P_1 .

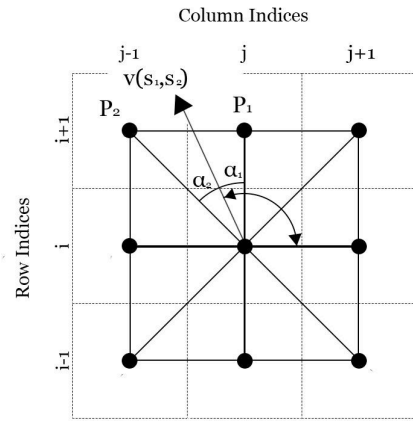


Fig. 3. Single direction flow direction D-Infinity

D. Cellular Automata

Cellular automata take the analogy of the appointment of his way to show all populations that interact in the cells, each cell is automata. By establishing rules approach into cellular automata, can be simulated through various circumstances which are complex, such as fluid motion introduced by *the Navier Stokes* [8]. Automata cellular make a big step to build a system that has a lot of objects as well as a lot of states from time to time [9]. A cellular automata is a model of the system "cell" object with the following characteristics.

- Cells displayed in grid.



Fig. 4. A simple form of a one-dimensional grid.

- Each cell has a state. Total state usually limited. A simple example, suppose two possibility of 1 and 0 (also exemplified in the form of "on" and "off" or "live" and "dead") [9]. State is a variable given to differentiate each cell. State can be a number or property. For instant each cell is expressed as part of a landscape, then the state is known as the number of individuals at each location or type of area that grows [8].

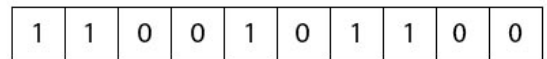


Fig. 5. A simple form of state.

- Each cell has a neighborhood. It can be defined in various ways. Usually the neighborhood is defined by a list of adjacent cells with the cell [9]

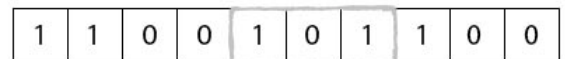


Fig. 6. Example of cells that have two adjacent neighbors: one on the left and one on the side right.

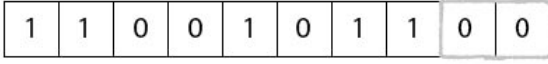


Fig. 7. Examples of cells that have an adjacent neighbor is on the left of the cell.

- The program, is a set of rules that are defined for changing state in response to state the current and its neighbors.

Formally, cellular automata is defined as a 5-tuple where $C = \langle S, s_0, G, d, f \rangle$, with: (1) S is the set of finite state (2) s_0 is the initial value of the state, s_0 element of S . (3) G is a neighboring cell. $G = \{i, i + r_1, i + r_2, \dots, i + r_n\}$ where n is the number of neighbors. (4) d is the dimension of the cell. (5) $f: S^n \rightarrow S$, the local rules. (6) $C(t)$ is the configuration at time t . $C(t) = (s_0(t), s_1(t), \dots, s_n(t))$ where N is the finite size of cellular automata and $s_i(t)$ is the state cell i at time t . (7) Global Mapping (F), $F: C(t) \rightarrow C(t+1)$. $C(t+1)$ is a tuple is the result of iteration t flow direction D-Infinity consisting of state himself and state neighboring cells.

III. RESULTS AND DISCUSSION

A. The Algorithm Flow Direction D-Infinity using Digital Elevation Model

In this study, as the initial map is a 3D map or a map with a three-dimensional coordinate system (x, y, z) , where x, y are the position values, and z as height values of a surface. The system made requires input in the form of raster data or maps in the form of a matrix (ASCII format).

Digital Map DEM TIFF format will be used in the algorithm flow direction D-Infinity and Cellular automata. Elevation in the map of DEM are represented by dots of pixels in the image, the image pixel map represents a single point of coordinates (x, y) which has a height value (z) .

The algorithm flow direction D-Infinity serves to determine the direction of water flow from one point where the water flow will move to one of the steepest facets steepest. While cellular automata serves to simulated process of water flow from the initial iteration until last iteration. Tarboton [1] provides a new method called flow direction with D-Infinity. It is said infinity because the direction of flow can be distributed to a value which is between $0 - 2\pi$. This method is calculated using eight triangular facets. Every facet has a vertex at the center pixel distributor and the other two are at the center of its neighboring pixels.

Upslope represents the vector (s_1, s_2) ,

$$s_1 = (e_0 - e_1) / d_1, \quad (1)$$

$$s_2 = (e_1 - e_2) / d_1, \quad (2)$$

where e_i and d_i is the value of elevation and distance between pixels in Figure 2.

Directions upslope and the amount given by the following equation.

$$r = \tan^{-1}(s_2 / s_1), s = (s_1^2 + s_2^2)^{1/2} \quad (3)$$

If r is not on $(0, 45$ degree), then r leads to one of the neighbors of the point of reviews on the facet and s is the amount of upslope are defined as follows:

$$\text{if } r < 0 \text{ then } r = 0, s = s_1 \quad (4)$$

if $r > 45$ then

$$r = 45, s = (e_0 - e_1) / (d_1^2 + d_2^2)^{1/2} \quad (5)$$

B. Cellular Automata

Formally flow direction D-Infinity model and water distribution into cellular automata are defined in the 5-tuple $C = \langle S, s_0, G, d, f \rangle$, where:

- $S = \{0,1\}$ where: 0 = status cells have no water discharge or equal to the height of the cells that have water discharge, 1 = status cells have water condition and drain some water to neighbors have lower elevation.
- s_0 is the initial state of the cell.
- $G = G(x', y') = \{(x', y') \mid x' = x + a, y' = y + b, (x', y') \neq (x, y), \forall a, b \in (-1, 0, 1)\}$, where $n = 8, r = 1$.
- d is rectangular two-dimensional.
- f is rule carried out in some time step until the water contained in the distributed pixels to pixels that are among the lowest or higher pixel so that water can no longer flow.

C. Simulation Results

Figure 8 and 9 displays solutions algorithm flow direction D-Infinity without looping (cellular automata). The yellow color in Figure 10 shows the lowest pixel is last of flow while the green is flow which be passed by water.

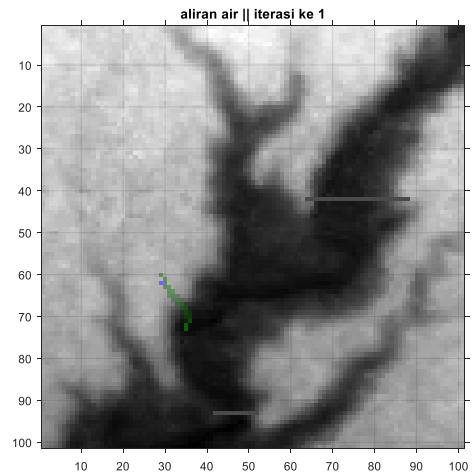


Fig. 8. The water flow of Kelara River map Jeneponto regency, South Sulawesi, Indonesia.

	29	30	31	32	33	34	35	36	37	38
60	0.33333	0	0	0	0	0	0	0	0	0
61	0	0.33333	0	0	0	0	0	0	0	0
62	1	0.33333	0	0	0	0	0	0	0	0
63	0	0.33333	0.33333	0	0	0	0	0	0	0
64	0	0	0.46989	0.19678	0	0	0	0	0	0
65	0	0	0.1925	0.33877	0.1354	0	0	0	0	0
66	0	0	0	0.1925	0.3544	0.11977	0	0	0	0
67	0	0	0	0	0.23747	0.34678	0.08241	0	0	0
68	0	0	0	0	0	0.30662	0.3114	0.04865	0	0
69	0	0	0	0	0	0.07706	0.33512	0.22576	0.02872	0
70	0	0	0	0	0	0.00753	0.2859	0.34451	0.01177	0.01695
71	0	0	0	0	0	0.00255	0.15237	0.48302	0.01177	0.01695
72	0	0	0	0	0	0	0.63795	0	0.02872	0.01695
73	0	0	0	0	0	0	0.63795	0.02872	0	0

Fig. 9. Water flow

	29	30	31	32	33	34	35	36	37	38
60	87.33333	88	90	91	89	83	76	73	69	60
61	92	89	89	90	90	85	78	72	63	51
62	89	89	90	88	86	83	80	74	56	40
63	90	89	88	87	83	82	78	70	53	37
64	89	89	86	85	82	80	75	66	51	37
65	88	87	82	80	77	73	70	61	49	39
66	88	84	79	76	71	66	58	50	40	32
67	84	79	74	68	60	51	42	36	31	26
68	81	74	66	57	50	40	32	27	24	23
69	81	73	62	50	41	31	25	21	18	17
70	80	72	58	44	35	24	18	16	16	15
71	76	68	56	42	29	17	13	11	11	11
72	75	68	53	40	21	13	9	11	9	9
73	74	67	53	36	19	12	8.637947	8.02872	10	12

Fig. 12. The elevation surface data

In the next iteration, elevation of surface land add with elevation of water surface (yellow) than do flow algorithm D-Infinity again to know the direction of water flow and so on until a predetermined iteration.

	29	30	31	32	33	34	35	36	37	38
60	0.33333	0	0	0	0	0	0	0	0	0
61	0	0.33333	0	0	0	0	0	0	0	0
62	1	0.33333	0	0	0	0	0	0	0	0
63	0	0.33333	0.33333	0	0	0	0	0	0	0
64	0	0	0.46989	0.19678	0	0	0	0	0	0
65	0	0	0.1925	0.33877	0.1354	0	0	0	0	0
66	0	0	0	0.1925	0.3544	0.11977	0	0	0	0
67	0	0	0	0	0.23747	0.34678	0.08241	0	0	0
68	0	0	0	0	0	0.30662	0.3114	0.04865	0	0
69	0	0	0	0	0	0.07706	0.33512	0.22576	0.02872	0
70	0	0	0	0	0	0.00753	0.2859	0.34451	0.01177	0.01695
71	0	0	0	0	0	0.00255	0.15237	0.48302	0.01177	0.01695
72	0	0	0	0	0	0	0.63795	0	0.02872	0.01695
73	0	0	0	0	0	0	0.63795	0.02872	0	0

Fig.10. Water Flow first iteration

The 3,000th iteration resulting water stop in the south and it rises to the north of the Kelara River. This simulation aims to see a prediction of water when adding the dam to the pixels 42.93 to 52.93, the water can be accommodated with a quantity of 3.267 million m³ with the assumption of the pixel width of 33 m and a rainfall of 1 m².

	29	30	31	32	33	34	35	36	37	38
60	87	88	90	91	89	83	76	73	69	60
61	92	89	89	90	90	85	78	72	63	51
62	89	89	90	88	86	83	80	74	56	40
63	90	89	88	87	83	82	78	70	53	37
64	89	89	86	85	82	80	75	66	51	37
65	88	87	82	80	77	73	70	61	49	39
66	88	84	79	76	71	66	58	50	40	32
67	84	79	74	68	60	51	42	36	31	26
68	81	74	66	57	50	40	32	27	24	23
69	81	73	62	50	41	31	25	21	18	17
70	80	72	58	44	35	24	18	16	16	15
71	76	68	56	42	29	17	13	11	11	11
72	75	68	53	40	21	13	9	11	9	9
73	74	67	53	36	19	12	8	8	10	12

Fig. 11. Elevation Data

	29	30	31	32	33	34	35	36	37	38
60	0.666667	0.083777	0	0	0	0	0	0	0	0
61	0	0.333333	0	0	0	0	0	0	0	0
62	1	0.333333	0	0	0	0	0	0	0	0
63	0	0.333333	0.333333	0	0	0	0	0	0	0
64	0	0	0.469889	0.196778	0	0	0	0	0	0
65	0	0	0.192497	0.33877	0.1354	0	0	0	0	0
66	0	0	0	0.192497	0.354399	0.11977	0	0	0	0
67	0	0	0	0	0	0.237471	0.346784	0.082412	0	0
68	0	0	0	0	0	0	0.306617	0.311399	0.048651	0
69	0	0	0	0	0	0	0.077063	0.335119	0.225765	0.02872
70	0	0	0	0	0	0	0.007533	0.285902	0.344511	0.011766
71	0	0	0	0	0	0	0.002554	0.152369	0.483024	0.011766
72	0	0	0	0	0	0	0	0.637947	0	0.02872
73	0	0	0	0	0	0	0	0.637947	0.695387	0

Fig. 13. Water flow second iteration

	29	30	31	32	33	34	35	36	37	38
60	87.66667	88	90	91	89	83	76	73	69	60
61	92	89	89	90	90	85	78	72	63	51
62	89	89	90	88	86	83	80	74	56	40
63	90	89	88	87	83	82	78	70	53	37
64	89	89	86	85	82	80	75	66	51	37
65	88	87	82	80	77	73	70	61	49	39
66	88	84	79	76	71	66	58	50	40	32
67	84	79	74	68	60	51	42	36	31	26
68	81	74	66	57	50	40	32	27	24	23
69	81	73	62	50	41	31	25	21	18	17
70	80	72	58	44	35	24	18	16	16	15
71	76	68	56	42	29	17	13	11	11	11
72	75	68	53	40	21	13	9	11	9	9
73	74	67	53	36	19	12	8.637947	8.695387	10	12

Fig. 14. The elevation surface second iteration

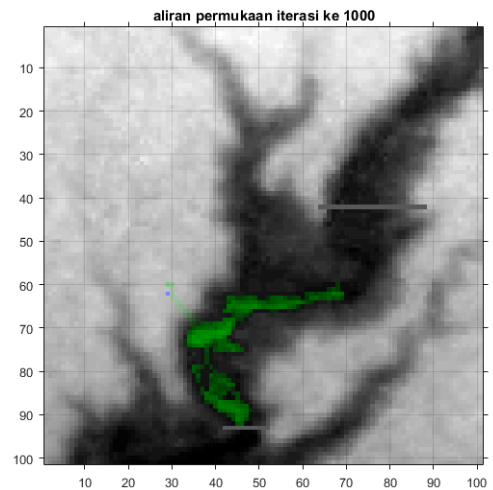


Fig. 15. Surface flow iteration 1000

IV. CONCLUSIONS

We have presented a realistic runoff simulation using cellular automata combined with the flow direction D -infinity as its transitions. The flow direction of the algorithm is not only influenced by surface slope but it is also the calculation resulted in the DEM values. After 5,000 iterations depicted in Figure 18, the dam could not withstand waterlogging. So that the water passes through the ground in the left side of the dam south through a height above the ground. The results of this simulation can be used as a basis for the authorities to consider dams' position to be constructed. In general, flow direction with finite iteration can be used as the guidelines for observing nature phenomena of runoff digitally.

For future work, we will investigate a simulation which includes velocity parameters so that a more accurate determination of the flow direction can be provided. Therefore, further research can pay attention to the velocity and soil absorption parameters to observe the quantity of water-flow and water-flooded naturally. Thus the problem of flooding in the rainy season can be anticipated early excessive in view of the ground surface flow phenomena.

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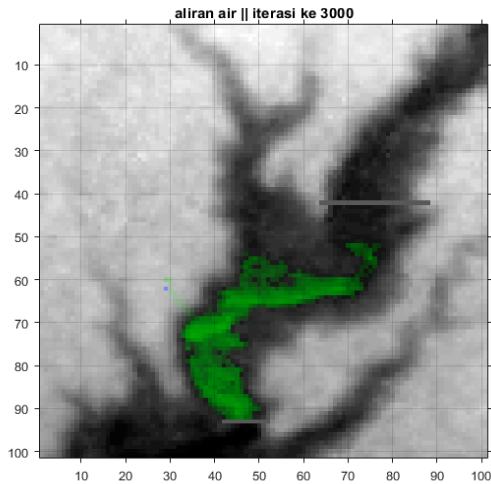


Fig. 16. Surface flow iteration 3000

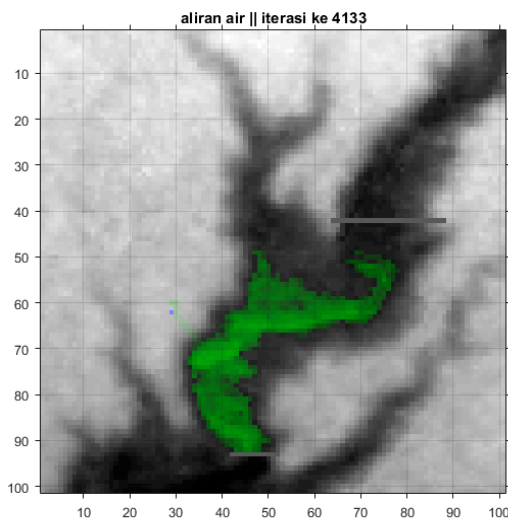


Fig. 17. Surface flow iteration 4133

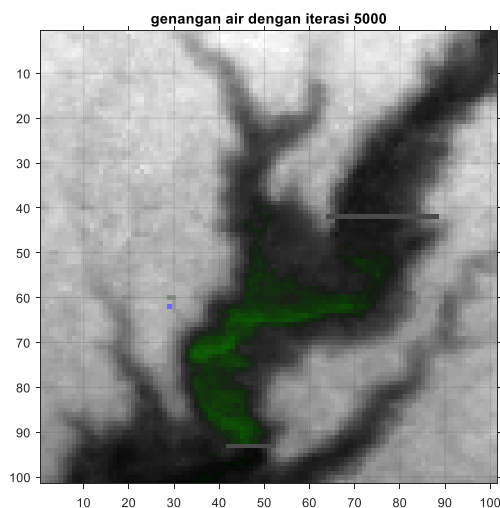


Fig. 18. Surface flow iteration 5000